On the arguments of O.E. Rössler

Introduction

Rössler argues that he is able to conclude, based on his reinterpretation of the Schwarzschild metric, that Black Holes do not emit Hawking radiation and will thus exist forever. Therefore, if produced at the LHC, they would represent a serious danger: instead of decaying by radiation, they would exist eternally and would have sufficient time to gradually devour their environment.

In what follows, we will sketch and analyze his argument. We will see that

- his argument concerns only the General Theory of Relativity (GRT), and makes no logical connection to LHC physics;
- the argument is not valid;
- the argument is not self-consistent.

The counterarguments presented here are independent from those discussed in the recent report by Giddings and Mangano, `arXiv:0806.3381[hep-ph]`. The present text should thus be seen as complementary to that report.

Background

In the following, let us repeat Rössler's mathematical arguments in a somewhat more compact notation.

The external Schwarzschild metric is ($r > 2m$)

$$g = -c^2 \left(1 - \tfrac{2m}{r}\right) dt^2 + \underbrace{\frac{dr^2}{1 - \frac{2m}{r}} + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)}_{h} \, . \tag{1}$$

Here, h represents the spatial component. The metric h defines the induced geometry on the spacelike hyperplanes $t = const.$

Light rays in the Schwarzschild geometry correspond to lightlike geodesics with respect to the space-time metric g. Since the latter is static (it possesses a timelike Killing field orthogonal to the hyperplane, represented here by $\partial/\partial(ct)$), the following well-known theorem holds: The light rays are unambiguously determined by their spatial projections (*i.e.* on any surface of $t = const.$), where in turn they are geodesics, but rather with respect to the *optical* (or *Fermat-*) *metric*. This metric is given by the spatial component of the Schwarzschild metric, divided by the squared absolute value of the static Killing vector field and thus by

$$\gamma = h/(1 - 2m/r)\,. \tag{2}$$

For instance, with this optical metric, the"optical distance" between two radially separated points r_u (up) and r_d (down) is given by

$$\int_{r_d}^{r_u} \frac{dr}{1 - \frac{2m}{r}}\,. \tag{3}$$

In contrast, the distance measured with h is

$$\int_{r_d}^{r_u} \frac{dr}{\sqrt{1 - \frac{2m}{r}}}\,. \tag{4}$$

The first integral diverges for $r_d \to 2m$. The "optical distance" to the horizon $r = 2m$ from each external point is thus infinitely large. We emphasize that this optical metric is the *spatial component* of a *space-time metric conformally rescaled by a factor* $1/(1 - 2m/r)$, which however neither fulfils the Einstein equations, nor has any physical relevance. Since it is conformally equivalent to the physical metric g, it does give identical results for the world lines of light and its time-of-flight retardation in gravitational fields (Shapiro effect). In contrast, it yields entirely wrong results when computing timelike geodesics (such as the trajectories of celestial bodies) which are in clear contradiction to established experimental results.

Rösslers "New idea"

With respect to the optical metric, this mathematical fact is uncontested. Yet, it is claimed by Rössler to be a new insight, though it has been well known to experts for a long time. It is a standard tool in the sense of an *auxiliary construction*, *e.g.* in the theory of gravitational lensing. In essence, Rösslers add-on is to promote this formal fact to a new physical principle and to say: *The optical distance is in fact equal to the real geometrical distance*.[1] He motivates this principle with the claim that the speed of light when defined *with respect to the optical distance* has the same value c everywhere, and relates it to a superficially similar claim made by Max Abraham in a dispute with Einstein in 1912, which however was based on

[1]We will not discuss here the fact that the word "real" is potentially void of content.

different assumptions and has become obsolete.[2] In this "optical geometry", bodies near the black hole horizon become arbitrarily large and the black hole itself cannot be understood any longer as a localizable object of finite extension.

An analogue from high school physics is geometrical optics in dispersive media. The medium is characterized by a space-dependent index of refraction $n(\vec{x})$, which is defined by the ratio of the speed of light in vacuum c to the locally variable phase velocity of light v_p: $n = c/v_p$. Also here, it is possible to define a new, "optical" geometry in space through the distance measure $ds_{\mathrm{optical}} = n\, ds_{\mathrm{euclidian}}$. This corresponds in turn to a conformal rescaling of the metric, this time by the factor n^2. By construction, the speed of light with respect to the new distance measure always equals c. According to Fermat's theorem, light rays are geodesics in the optical metric. In this view, the deflection of light in dispersive media is not a consequence of the locally variable speed of light but a deviation from Euclidian geometry. In this way, the spatial propagation of light rays can be determined with help of an analogous auxiliary construction, by fictitiously assigning an optical metric to space and determining geodesics with respect to this metric. However nobody would think of attributing to this "optical geometry" any other role than that of an auxiliary construction.

On the basis of Rössler's reinterpretation, his argument for the absence of Hawking radiation in the Schwarzschild space-time (1) simply reduces to the statement: what is infinitely far away, cannot radiate.

Inconsistencies

If we take all this seriously for a moment, we face immediately the following questions:

1. How can something that is *infinitely* far away (and also something that is infinitely large) be created in a *finite* amount of time, and have an effect on us? Should Rössler not conclude in the same way that Black Holes cannot be created in the first place? But what about astronomical data showing signatures of black holes, *e.g.* in the center of our galaxy?

2. The energies of the cosmic rays that constantly bombard our atmosphere are (after transformation into the center-of-mass system) many orders of magnitude higher than the energies that can be reached with the LHC. However we have no hints that they produce Black Holes – why not? A possible answer

[2] It is obsolete because it was related to the hypothesis of a *scalar* (spin = 0) theory of the gravitational field in the framework of the *special* theory of relativity. Today it is known that such a theory exists mathematically but leads to wrong predictions, such as the absence of deflection of light in gravitational fields, and a *retrograde* perihelion rotation of only $1/6$ of the Einstein value.

could be that on the earth we are not in the center-of-mass system of the cosmic radiation. A Black Hole produced by cosmic rays would traverse the earth at high speed, and would thus have no time to cause any damage. This requires however the Black Hole to be a *localizable object in space*, from which we can be more or less far away. However in Rössler's interpretation this is exactly not the case, since the Black Hole is always infinitely far away from us. Therefore, the answer that is meaningful in the framework of the usual interpretation no longer makes sense in Rössler's interpretation.

Summary

It is well known and uncontested that the optical metric is a structural ingredient of GRT, with the meaning of an auxiliary construction, which has proved especially successful when computing the trajectories of light rays in space. In the same way, it can even be employed for the effective calculation of some properties of the movement of massive bodies (*e.g.* the computation of the centrifugal force). However it is unequivocally wrong and meaningless to claim, as Rössler does, that it would be the only geometrical framework that governs every physical process. For example, it would lead to predictions for the planetary orbits that are badly wrong. Rössler's arguments thus rest on a generalization of optical geometry that is not only unfounded but demonstrably wrong. Therefore, it may not be surprising that beyond his ad-hoc assertion, he is unable to construct a reproducible, logical connection between optical geometry and Hawking radiation. Moreover, such a connection would have to conform to fundamental principles of GRT, such as the invariance of physical laws under any choice of local coordinates: the prediction that a black hole radiates does not depend on any particular notion of spatial distance in the space-time geometry determined by the Schwarzschild solution.

Moreover, and as was shown above, Rössler's arguments are not even self-consistent.

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